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# Why does the Secchi disk disappear? An imaging perspective

Weilin Hou, Zhongping Lee, Alan D. Weidemann

Naval Research Laboratory, Code 7333, Stennis Space Center, MS 39529  
[hou@nrlssc.navy.mil](mailto:hou@nrlssc.navy.mil)

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## 1. Introduction

Ocean optics can trace its origin to understanding and determining underwater visibility. The simple, widely-used Secchi disk method, dating back to 1865, is still used by biologists and physicists as a quick measurement of water clarity in natural waters. Its deployment is simple: one lowers the traditionally white, circular disk about 30cm in diameter, from above the water into the water column, and determines the point at which it disappears from sight. The depth at which the disk can no longer be discerned against the background is defined as the Secchi disk depth ( $Z_{SD}$ ). Measurement protocols include measurement of the depth at which the disk disappears, and then re-appears and then using the average of both values to eliminate errors imposed by possible residual memory effects, and by different operators. Secchi depth is often used as a convenient way of quantifying water clarity. It could also be used horizontally, to help measure diver visibilities, or to avoid issues associated with its vertical deployment (ie sunglint, surface waves, bottom patchiness etc), especially in very shallow areas. Preisendorfer [1, 2] thoroughly summarized and reviewed the science behind the Secchi disk approach from the radiative transfer perspective, and warned against potential over-usages of its application in what he termed "Secchi Disk madness." Even though new, more rigorous electro-optical instruments are available to measure optical properties, the Secchi depth still has proven to be an inexpensive, dependable, and efficient measure of the water clarity that has been related to diver visibility, water quality, and biological activities [2, 3]. The vast database of available Secchi depth accumulated over the years throughout the entire world, nonetheless, adds the incentive to better understand the science behind this simple approach. The implications of the Secchi disk method on underwater imaging issues such as prediction of diver visibility for mine identification also prompt further study beyond the traditional radiative transfer approach.

For most theoretical treatments of Secchi disk visibility, the common definition of the visibility contrast (Weber contrast) is used [1, 2, 4]. We shall follow conventional notations cited and briefly outline the theory. Details can be found in above mentioned references. The visibility contrast  $C_v$  is given as the normalized difference in brightness (radiance,  $L(\theta, \phi, z)$ ) between the target (Secchi disk)

$$C_v = \frac{L_T(\theta, \phi, z) - L_B(\theta, \phi, z)}{L_B(\theta, \phi, z)}, \quad (1)$$

where subscripts T and B denote the target (disk) and the background respectively. As the viewing range decreases to zero (without the medium attenuation and scattering), the above gives the inherent contrast ( $C_{v0}$ ). For horizontal orientation and viewing, as the disk is moved further away (increasing  $z$ ) it can be shown that the apparent contrast ( $C_{vz}$ ) decreases exponentially as function of the medium's attenuation [2, 5]:

$$C_{vz} = C_{v0} e^{-cz}, \quad (2)$$

where  $c$  is the photopic beam attenuation coefficient, which is the attenuation of the natural light spectrum convolved with the spectral responsivity of the human eye (photopic response function). For most viewing conditions this can be approximated by a monochromatic beam attenuation near the peak of the human eye sensitivity such as at 532nm [4]. When the disk is lowered vertically into the water column, one must take into account that the illumination at the disk and background is being influenced by the surface radiance and attenuation of light, therefore the above equation is modified by the diffuse attenuation, and becomes [2, 5]

$$C_{vz} = C_{v0} e^{-(c+K)z}, \quad (3)$$

where  $K$  is the downward diffuse attenuation coefficient (ignoring the viewing angle dependence for simplicity). The above (Eq. (3)) is what Preisendorfer referred to as the basic equation of Secchi disk science. Or in a different form when the contrast of the target and background converge

$$c + K = -\frac{\ln(C_L)}{Z_{SD}}, \quad C_L = \left| \frac{C_{vz}}{C_{v0}} \right|, \quad (4)$$

where  $C_L$  is the limiting contrast at disappearing disk depth ( $Z_{SD}$ ) [4].  $C_L$  has been shown to be a function of the disk size, but more importantly the adaptation luminance at the disk location [6]. The contrast threshold has been found to vary from 0.02 or higher with low adaptation luminance, to 0.008 [7] or even 0.002 at high luminance [6]. Unless mentioned, all parameters are associated with photopic quantities hereon. It can be shown that such simplification does not invalidate derivations that will follow [4]. The spectral and angular dependence of radiance distribution, as well as the effects of surface glint, are also neglected for simplicity.

When a black Secchi disk is used instead of the traditional white one, studies suggest that the same theory holds with less variability [7], and therefore the black disk can be used as a robust measure of underwater visibility [4]. Other modifications of the disk, such as one with alternating black and white quadrants, are also used by many researchers, especially those monitoring lakes. Results show visibility ranges measured with a white/black quadrant disk are similar to those obtained with the single color black disk [8]. This has important implications from the imaging perspective in that the white/black quadrant disk can be viewed as a pseudo-resolution chart, with a separation distance about half the size of the disk diameter. This makes sense if we consider a disk with half of its side painted black and half painted white, which can be approximated to a bar pattern with a cycle of  $d$ . Splitting the area again to the quadrant Secchi disk reduces the size of the pattern by half to approximately a  $d/2$  cycle.

Intuitively, since the disappearance of Secchi disk is observed by a human operator, it is fitting to examine the issue from the imaging perspective. In this regard, underwater visibility can be thought of as the resolution reduction of specific spatial frequencies over distance. This is usually studied by examining a standard resolution chart such as USAF 1951 and known environmental transmittance characteristics. To understand how the visibility contrast fits the image theory, it is prudent to review the background briefly. Details can be found in many image processing books such as [9].

Generally speaking, a 2-dimensional image of an object is basically the combination of original signal,  $f(x,y)$ , convolved by the imaging system response  $h(x,y)$ , integrated over sensor space  $\Xi$ :

$$g(x, y) = \iint_{\Xi} f(x_i, y_i) h(x - x_i, y - y_i) dx_i dy_i, \quad (5)$$

or,

$$g(x, y) = f(x, y) \otimes h(x, y), \quad (6)$$

here  $\otimes$  denotes 2-D convolution, and  $h(x,y)$  is the system response to a point source, or the point-spread function (PSF). The system response includes those from both the imaging system itself, as well as the effects of the medium (water in our case).

Mathematically, it is easier to manipulate the above relationship in the frequency domain as the convolution operator becomes simple multiplication. Applying Fourier transform, the above relationship becomes



$$G(u, v) = F(u, v)H(u, v), \quad (7)$$

where  $u, v$  are spatial frequencies and  $G, F, H$  are Fourier transforms of  $g, f$  and  $h$  respectively. The Fourier transfer of  $h$ , for example, is in the following form:

$$H(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y) e^{-j2\pi(ux+vy)} dx dy, \quad (8)$$

The system response function  $H$ , also referred to as the optical transfer function (OTF), is the Fourier transform of the PSF. The magnitude of the OTF is the modulation transfer function (MTF). The MTF describes the contrast response of a system at different spatial frequencies, and when the phase information is of little concern, it is a sufficient measure of the power transfer. By definition, therefore, the MTF can be measured by the contrast of sinusoid or bar patterns of corresponding spatial frequencies [9, 10], and is the focus of this study.

Notice that the above MTF term  $H(u, v)$  is the total system response. Therefore if one views the complete path from target to the bottom of eyes or the recording CCD plane, in many cases, the MTF can be the effect of multiple individual components. In the frequency domain, the MTF can be expressed by the direct product of each component, for instance, the optical system itself, and the medium (plus any other factors when applicable):

$$H(u, v) = H_{\text{system}}(u, v)H_{\text{medium}}(u, v). \quad (9)$$

The system response  $H_{\text{system}}(u, v)$  can be pre-determined and calibrated to remove any significant errors, and in most cases, does not vary with imaging conditions. The above formulation, which emphasizes the validity of the separation of the system and the medium, is significant in our analysis. Furthermore, one should pay special attention to the band-limiting characteristics imposed by  $H_{\text{system}}$ , such as a camera system's field-of-view, and Nyquist sampling frequency limits imposed by the CCD resolution [9]. For a known object, by examining the characteristics of the medium evident in  $H_{\text{medium}}(u, v)$ , one could theoretically predict the exact outcome image within that environment by responses from all of the individual spatial frequencies. In contrast, one can inversely derive the detailed information of the object from the outcome image via inverse Fourier transform, which is the goal of underwater mine detection and other applications including target recognition and tracking. Needless to say, the presence of various noises (such as scattering or surface fluctuations) complicates these through-the-sensor techniques and is beyond the scope of this paper. Instead, the focus here is on the optical properties which influence the modulation transfer functions for the underwater environments as they relate to the disappearance of the Secchi disk. To better illustrate the main problem from the imaging point of view, we choose optimal viewing conditions associated with Secchi depth measurement: minimal influence from surface waves, sunglint, shadows, and bottom reflectance. For convenience we focus on the horizontal visibility range with  $Z_{SD}$  used to generally refer to the disappearance range. While we focus on the horizontal case, the vertical depth can be derived similarly from the horizontal results by simply adding photopic diffuse attenuation  $K$  to the  $c$  values Eqs. (2)(3).

## 2. Secchi theory based on modulation transfer

Recall that the modulation transfer function measures the modulation or contrast of a pattern after transmission through the system. The modulation can be obtained by measuring maximum and minimum values of the sinusoid (or bar) pattern of spatial frequency  $\psi$  [9, 10],

$$M(\psi, z) = \frac{S_{\max}(\psi, z) - S_{\min}(\psi, z)}{S_{\max}(\psi, z) + S_{\min}(\psi, z)}, \quad (10)$$

where  $S_{\max}$  and  $S_{\min}$  denote the maximum and minimum “brightness” values of the pattern at each spatial frequency  $\psi$ , at range  $Z$ . These terms can be related to the luminance level of the image, or the radiance detected by the image sensor.

By normalizing to the input or initial modulation (assuming uniform), which is

$$M(\psi, 0) = \frac{S_{\max}(\psi, 0) - S_{\min}(\psi, 0)}{S_{\max}(\psi, 0) + S_{\min}(\psi, 0)} = M_0, \quad (11)$$

the modulation transfer function  $H(\psi, Z)$  can be expressed as

$$H(\psi, z) = \frac{M(\psi, z)}{M_0}. \quad (12)$$

When considering black versus white patterns the source modulation is unity,  $M_0 (=1)$  can be omitted. Operationally, normalization can be applied by dividing the low frequency value of the system  $M_0$  before the transfer. Note that while the above contrast definition (also known as Michelson contrast) is different from the traditional contrast equation used in Secchi disk theory, when the Secchi disk disappears the information content is the same in both formulations.

For circular symmetrical response systems, such as the isotropic volume scattering type found in the seawater, the corresponding 2-dimensional transforms found in Eq. (8) can be reduced to a one-dimensional Hankel (Fourier-Bessel) integral, such as for  $H$ :

$$H(\psi, z) = 2\pi \int_{\theta=0}^{\theta_{\max}} J_0(2\pi\theta\psi) h(\theta, z) \theta d\theta. \quad (13)$$

Wells [11] applied small angle approximations to the above and derived a robust underwater modulation transfer model which is briefly outlined below. By separating the exponential decay effect with distance due to the medium, the MTF of the medium in Eq. (13) can be expressed as

$$H(\psi, z) = e^{-D(\psi)Z}, \quad (14)$$

where  $D(\psi)$  is the decay transfer function (DTF) and is independent of the range of detection. This provides a method to compare measurements at different ranges for consistency.

By using a thin slab model with the small angle scattering approximation, the decay function can be written as the photopic beam attenuation  $c$ , less the light scattered back into the acceptance cone, which is

$$D(\psi) = c - S(\psi), \quad (15)$$

where

$$S(\psi) = 2\pi \int_{\theta=0}^{\theta_{\max}} J_0(2\pi\theta\psi) \beta(\theta) \theta d\theta, \quad (16)$$

and  $\beta(\theta)$  is the volume scattering function. Total scattering coefficient  $b$  is obtained via

$$b = 2\pi \int_0^\pi \beta(\theta) \sin \theta d\theta. \quad (17)$$

In an effort to derive a closed-form solution, Wells [11] assumed a scattering function with the following analytical form,



$$\beta(\theta) = \frac{b\theta_0}{2\pi(\theta_0^2 + \theta^2)^{3/2}}, \quad (18)$$

where  $\theta_0$  is related to the mean square angle (MSA). When compared with coastal measurements by Petzold [12] (Fig. 1), this function approximates the behavior of the scattering in small angles reasonably well in most cases. More discussion about these curves will follow in the next section. Combining Eqs. (15)-(18), the DTF of the seawater can be expressed as

$$\begin{aligned} D(\psi) &= c - S(\psi) \\ &= c - \frac{b(1 - e^{-2\pi\theta_0\psi})}{2\pi\theta_0\psi}. \end{aligned} \quad (19)$$

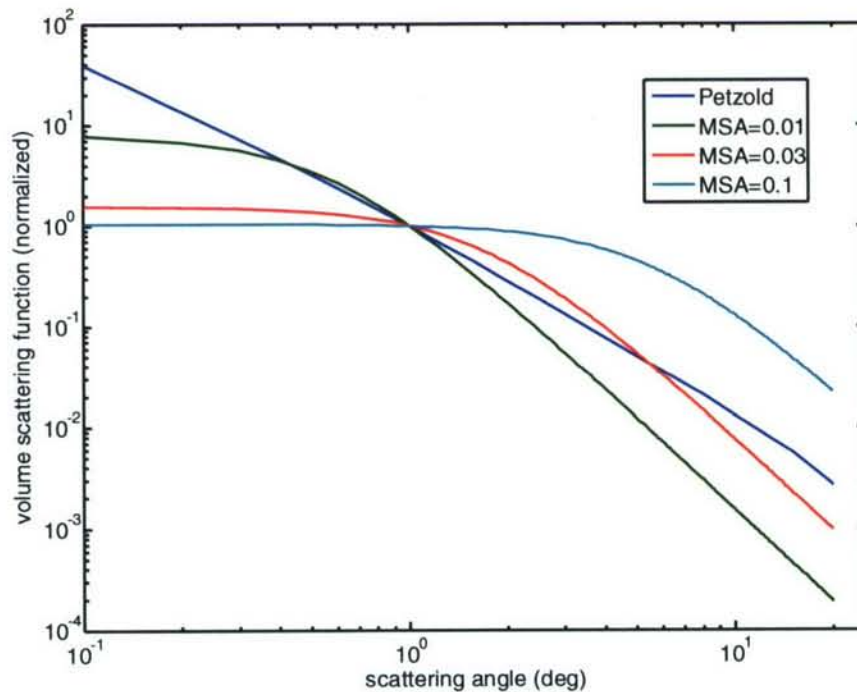


Fig. 1. Well's phase function Eq. (18) with different MSA parameters compared to measured coastal water phase function by Petzold (reproduced from Table 3.10 of [12]), up to 20°. All curves are normalized at 1° for comparison.

Equations (14) and (19) above reveal the relationship between the water MTF, the total attenuation, the spatial frequencies involved (which can be related to the physical dimensions of the target or Secchi disk), and the depth or horizontal range at which the Secchi disk disappears. For example, assume single scattering albedo  $\omega_0 = b/c = 0.8$ ,  $\theta_0 = 0.03$ ,  $cZ = 4.8$ , the MTF of the water plotted in Fig. 2 shows a large and rapid reduction with increasing angular frequencies, following Eq. (12). It is clearly shown that finer details (higher frequencies) are lost first. Notice that the term  $e^{-2\pi\theta_0\psi}$  only affects very low spatial frequencies.

From a qualitative MTF perspective it is clear that the disappearance of the Secchi disk as it is moved away from the observer is the result of reduced resolution by the system response

function, or MTF, at the spatial frequency related to the disk size and range. Here for simplicity, we exclude the effect of imaging system itself (the human eye in this situation), as well as that of the air-sea interface when considering vertical application, and consider only the effect of water transmission.

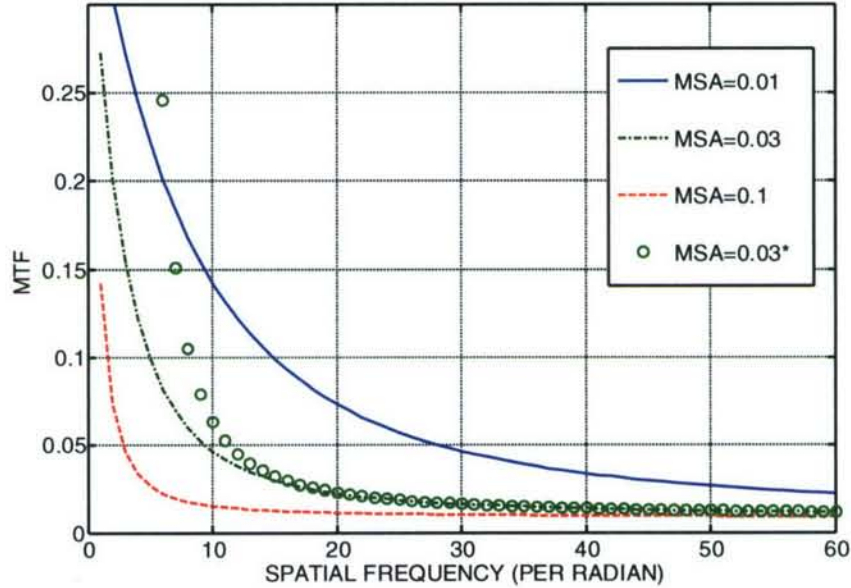


Fig. 2. MTF of water at different mean square scattering settings Eq. (19), with optical length  $DZ=4.8$ . The curve with circles demonstrate the MTF without  $e^{-2\pi\theta_0\psi}$  term in Eq. (19). See text for details.

When the limiting contrast threshold  $C_L$  is reached, the Secchi disk can no longer be seen by the human operator. The resulting effective spatial frequency ( $\psi_{SD}$ ) reflects all frequencies related to the disappearance of the disk. These include those related to the actual disk size (low frequencies) extending to those associated with the disk edge (high frequencies). Recall our earlier discussion regarding the alternating black and white Secchi disk as a pseudo resolution chart with a separation distance of  $d/2$ , the first order approach to approximate  $\psi_{SD}$  can be expressed as

$$\psi_{SD} = Z_{SD} / (d/2) = 2Z_{SD} / d. \quad (20)$$

Another way to interpret the above equation is that as the disk is moved away from the observer, the corresponding spatial frequency increases (ie, narrower subtense angle). From Fig.2, we see that due to the rapid decay of modulation with increasing frequencies, any errors introduced by above approximation will also decrease rapidly when the disk is moving away from the observer.

Assume at this point that the difference between the target brightness and the background is small (but not zero), such that (dropping angular notations hereon for simplicity)

$$L_T \approx L_B \text{ or } C_L \ll 1. \quad (21)$$

We can relate the visibility contrast to the modulation transfer function via the following:



$$\begin{aligned}
 H(\psi_{SD}) &= \left[ \frac{S_{\max}(\psi_{SD}) - S_{\min}(\psi_{SD})}{S_{\max}(\psi_{SD}) + S_{\min}(\psi_{SD})} \right] / M_0 \\
 &= \left( \frac{L_T - L_B}{L_T + L_B} \right) / M_0 = \frac{C_L}{(C_L + 2)} / M_0 \approx \frac{C_L}{2M_0}.
 \end{aligned} \quad (22)$$

As noted previously since both contrast terms depict the same target, the informational content is the same. The difference exists only in mathematical forms, yet it gives a new perspective about Secchi disk disappearance. It is easy to see from Eq. (22) that for  $C_L=0.02$ , the above approximation introduces about 1% error.

Rewriting Eq. (14),

$$D(\psi) = -\frac{\ln[H(\psi)]}{R}. \quad (23)$$

At Secchi depth  $Z = Z_{SD}$ , combining Eqs. (19), (22) and (23):

$$c - \frac{b(1 - e^{-2\pi\theta_0\psi_{SD}})}{2\pi\theta_0\psi_{SD}} = -\frac{1}{Z_{SD}} \ln\left(\frac{C_L}{2M_0}\right). \quad (24)$$

For the conditions  $\theta_0 \sim 0.03$  [13], disk size  $d=0.3\text{m}$ , for  $Z_{SD}$  on the order of 10m,  $\exp(-2\pi\psi_{SD}\theta_0) \sim 0.002 \ll 1$ . Let  $\varsigma = -\ln(C_L/2M_0)$ , then

$$c - \frac{b}{2\pi\theta_0\psi_{SD}} = \frac{\varsigma}{Z_{SD}}. \quad (25)$$

The above is the derived horizontal visibility range. For vertical Secchi disk applications, one can show from Eqs. (2), (3) that

$$c + K - \frac{b}{2\pi\theta_0\psi_{SD}} = \frac{\varsigma}{Z_{SD}}. \quad (26)$$

Eq. (26) is the equivalent form of the basic equation for Secchi disk referred to by Preisendorfer [2], but now derived using modulation transfer theory. If we let  $\Gamma = \varsigma + db/(4\pi\theta_0)$ , then the Secchi disk depth (or visibility range in horizontal direction) can be written as

$$cZ_{SD} = \Gamma \quad \text{for the horizontal case, and} \quad (27)$$

$$(c + K)Z_{SD} = \Gamma \quad \text{for the vertical case.} \quad (28)$$

These have the same form and are directly comparable to the radiative transfer results [2].

Rewriting Eq. (27) gives the horizontal visibility range as

$$Z_{SD} = \frac{\Gamma}{c}. \quad (29)$$

Now let's consider a situation where the black Secchi disk is used. Take the generally accepted limiting contrast as 0.02 from atmospheric studies [14], with  $M_0=1$ ,  $C_0=1$ ,  $\varsigma$  is then 4.6. Assuming the total scattering is  $0.2 \text{ m}^{-1}$  with  $\theta_0=0.03$ , then for a 30cm Secchi disk, the

second term in  $\Gamma$  is given as  $db/4\pi\theta_0c \sim 0.16/c$ . Therefore the coupling constant  $\Gamma$  is 4.76. The horizontal visibility range for the black Secchi disk therefore becomes

$$\text{visibility} \sim \frac{4.8}{c}, \quad (30)$$

which is inline with previous studies [4].

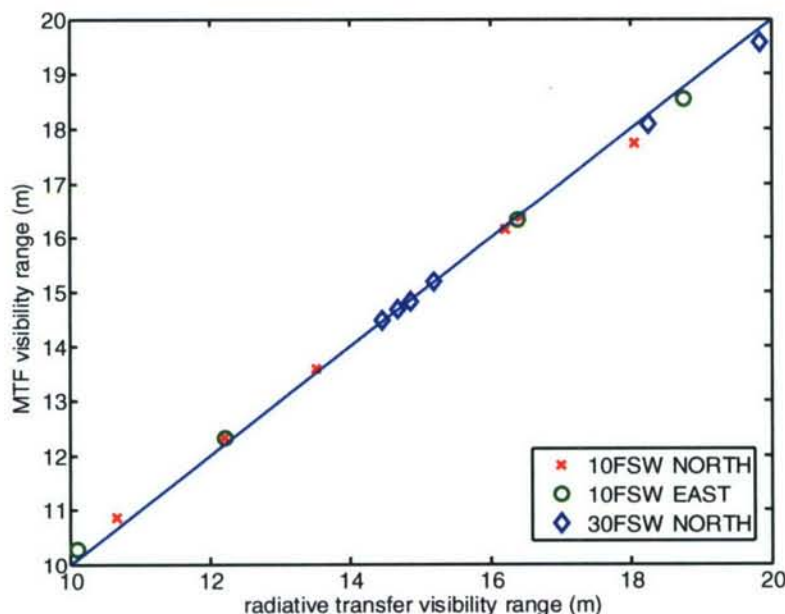


Fig. 3. Visibility range comparisons between previous radiative transfer approach and current MTF approach.  $c(532)$  ranges from 0.24 to 0.45. See text for details.

Using absorption and attenuation coefficients measured with an ac-9 (WetLabs Inc) during the GLOW experiment (Gauging Littoral Optics for the Warfighter, Sept 17-22, 2001, Pensacola, FL), calculated visibility ranges using the radiative transfer and MTF approaches are shown in Fig. 3. Visibility measurements were taken at 10-foot (10FSW) and 30-foot (30FSW) water depths (depth below the surface) each day, following the same track directions each day. Attenuation ( $c$ ) and scattering ( $b$ ) values at 532nm are used to represent photopic attenuation and scattering [4] for both approaches. Each data point represents the averaged value at the specific depth on each day.  $4.8/c$  is used to derive visibility ranges representing previous studies [4], while Eq. (29) is used to obtain MTF visibility ranges along with typical parameters ( $\theta_0=0.03$ ,  $d=0.3\text{m}$ ). The two approaches generally agree well as shown. However the current approach gives a smaller predicted visibility range in clearer waters ( $c\sim 0.24$ ) and a larger range in more turbid waters ( $c\sim 0.45$ ). The solid line depicts 1:1 ratio.

### 3. Discussion

To the authors' knowledge, the effect of disk size on Secchi depth has only been implicitly embedded in previous theoretical treatments of the Secchi disk method, usually with respect to the subtense angle for the eye. In other words, the traditional theory based on the radiative transfer approach applies regardless of whether the disk is 1mm or 10m in diameter. Using simple but important assumptions, this study demonstrates a similar Secchi depth/visibility



range can be reached using a more general approach involving all spatial frequencies. In doing so, the Secchi depth is now explicitly related to the disk size as well as the scattering. The decay of MTF at spatial frequencies over the observing range (Fig. 2) provides the straightforward answer to the Secchi disk visibility: as the disk moves away from the observer, the spatial frequency corresponding to the disk size increases, and the MTF decreases as a result of both absorption and scattering at an increased rate. From Eq. (19), we see that the role of the total attenuation ( $c$ ) is to dampen the MTF by the same amount to all frequencies, which is the basis of the previous radiative transfer approach. The current MTF-based approach shows, on the other hand, that the scattering does not affect all frequencies equally Eq. (19), (Fig. 2). The only time both approaches would converge is when the second term on the right side of Eq. (19) becomes insignificant. This is precisely the case for the Secchi disk where the total attenuation plays the dominant role at increased higher spatial frequencies by reducing the target contrast. This explains why the original radiative transfer approach by Preisendorfer and Duntley works so well.

Throughout the history of Secchi disk usage various disk sizes have been used ranging from a few centimeters to meters in diameter. The disk sizes most often used are between 20 and 40cm in diameter; with the 30cm being the standard for marine scientists, while lake researchers seem to prefer the 20cm size. Radiative transfer theory results do not explicitly contain relationships between the Secchi depth and the disk size. Rather, it is embedded implicitly in the contrast threshold  $C_v$ . From the MTF aspect, however, the functional dependence of Secchi disk visibility and disk size, and the relationship with forward scattering Eqs. (27) (28), is explicit in the formulation. There are other contributors to a change in the apparent size of the disk such as deployment height and viewing angle, but here we address disk size specifically.

Because of the large range of disk sizes used it is important to examine the impact disk size has on the overall visibility range. We can derive the rate of variation of  $Z_{SD}$  as a function of the disk size using Eq. (28) and demonstrate that

$$\frac{\partial Z_{SD}}{\partial d} = \frac{b}{4\pi\theta_0(c+K)} \quad (31)$$

Applying typical values found in coastal regions [15] (ie  $a=0.14$ ,  $b=0.18$  for bay, and  $a=0.041$ ,  $b=0.21$  for coastal waters, respectively), the change in the range would be 3~5% for waters with a small MSA value (0.03), and 1~1.6% for a larger MSA value (0.1). Clearly the rate of variation has a relatively small contribution to the overall visibility range, as it only affects a small portion of  $\Gamma$ . Likewise, using typical values above, it can be seen that the scattering term represents roughly 3% of the variation in  $\Gamma$ . One may also notice that the approximation error of  $\Psi_{SD}$  in Eq. (20) does little to the outcome of the current theory. For example, if a cycle of  $d$  instead of  $d/2$  is used, the change in  $\Gamma$  is only  $0.08/c$  for Eq. (30). Together, these further help explain the convergence of radiative transfer and the MTF approaches.

Blackwell also showed [6] that the limiting contrast  $C_L$  is a function of disk size (or angular subtense) and adaptation luminance, ie,  $\partial C_L / \partial d \neq 0$ . Analysis with the radiative transfer method has shown a small (<1%) change in range visibility due to disk size changes (43cm to 60cm) [1]. This effect can also be explained by Fig.2. From the modulation transfer perspective it can be seen that the contrast decay of the spatial frequency reaches a plateau at higher spatial frequencies. In other words, due to the fast decay in the MTF it does not really matter much if the disk size is 20cm or 30cm. This is an impact only when  $Z_{SD}$  is very small or when the disk is very large (ie low frequencies). This rapid decay is the key reason behind observations that the target size does not significantly alter the disappearance range of the disk. The further away the disk is, the higher the spatial frequencies it corresponds to, and where the MTF at these frequencies flattened out is about the same for the 30cm and the 20cm disk.



As for disk color, from the modulation transfer theory Eqs. (11) (25), and everything else being equal, the usage of the black disk versus black and white alternating disk should have the same outcome since they both have  $M_0=1$ . This has been confirmed by daily Secchi depth measurements over a 6 month time span done in Skagit River [8]. Therefore, both types of disks are equally useful when measuring underwater visibility and water quality [4].

The contrast term,  $\zeta$ , is somewhat less defined since it is solely dependent upon each observer's ability to discern contrast, and subsequently is also a function of conditions at which measurements are taken (ie, ambient light, sea state, surface glint, sun angle, shadow or sunny side of vessel, disk types, bottom types, and so on). However, claims have been made to indicate very small variations (<1%) among different observer groups, volunteers versus professionals amongst lake monitors in Florida [16]. This result seems inconsistent with the theory which suggests that for differing background adaptation luminance, combined with different contrast values (0.02 to 0.005), the possible  $\Gamma$  values can range from 5.9 to 9.3 for a white disk with 85% reflectance, even under optimal observation conditions, per Preisendorfer [2]. Using typical coastal region parameters quoted earlier, the results here show  $\Gamma$  ranging from 6.5 to 10.1 Eq. (27) under similar conditions as Preisendorfer [2]. Other studies report different values, such as  $\Gamma=8.68$  [17], and  $\Gamma=4.8$  [4]. As expected the  $\Gamma$  value can be significantly higher under conditions of strong scattering. For example, when  $b=2$  and all other parameters stay the same as in the earlier black disk case (where  $\Gamma=4.76$  ( $b=0.2$ )) then  $\Gamma=6.2$  will be obtained with the current formulation. The larger range of  $\Gamma$  values can be explained, in part, by the differences in the scattering contribution to the total attenuation.

The modulation transfer method presented here has the benefit of a general approach, and is applicable to other underwater visibility issues including self-illuminating targets. The disappearing frequency corresponds to any contrast related features, such as patterns and textures, so long as the dependant spatial frequency is not so small that it is limited by the system hardware before the medium Eq. (9). The only difference when applying the theory to different targets is in the threshold contrast that is required.

The small angle approximation by Wells is applicable to most vision and imaging systems with a field-of-view cone less than  $20^\circ$ . It considers photons scattered within this angle as part of the signal to be included. This inherently assumes strong forward scattering in the medium. Monte Carlo simulation results indicate that at the typical spatial frequency for a Secchi disk range (>50 cycles per radian), the small angle approximation error is about 2% for up to 10 attenuation lengths [15]. In addition, studies also show that the exact shape of the volume scattering function Eq. (18) does not affect the outcome significantly (<1%), as it has little impact on the cumulative effect of PSF, and therefore the MTF [13]. However, departure from the small angle approximation assumption will invalidate the modulation transfer theory applied here. For instance, the theory is not valid for a Raleigh-type scattering medium.

The assumption to eliminate the  $\exp(-2\pi\psi_{SD}\theta_0)$  term in Eq. (24) may be invalid in very turbid environments. For example, if visibility is only 1.5m,  $\psi_{SD}=10$  and  $\exp(-2\pi\psi_{SD}\theta_0)=0.15$ , in which case the contribution should be included as it represents ~ 15% variation. However, from discussions above, since this entire term contributes to only ~3% to  $\Gamma$  overall, the conclusions arrived at in Eqs. (26)-(29) still hold well. Further, a closer examination at different spatial frequencies reveals that this term would affect only low frequencies (Fig. 2), and therefore is not a major concern in the case of the Secchi disk.

Modulation transfer decay (blur) due to the turbulence of the medium is not accounted for in our discussion, nor is other factors that reduce visibility such as surface glint and capillary waves. In addition, one cannot ignore an implicit assumption when applying modulation transfer theories, namely the shift invariance of the system. The shift invariance roughly translates into requirements in the field that Secchi disk irradiance does not change over the period of observation. Fortunately, the human vision system automatically adapts and eliminates many of these effects. This requirement is important, however, when electro-optic imaging systems are used or designed based on the above discussed theory.



#### **4. Conclusion**

The disappearance of the Secchi disk is the result of the reduced modulation contrast of the disk in the water medium. This is brought about by absorption and scattering, with the latter playing a more significant role due to its effect on the contrast reduction. We demonstrate that by applying the modulation transfer theory, the Secchi depth and what is generally referred to as the horizontal visibility can be found using a more general approach involving all spatial frequencies. We show that, when typical optical properties are used, predicted visibility ranges are comparable to those obtained using radiative transfer methods. This result is obtained under the assumption that the small angle scattering approximation is valid, which demands strong forward scattering as the pre-condition. The effect of disk size to Secchi depth is included in the current form, both implicitly embedded in contrast threshold as previous studies, but also explicitly expressed as well. The improvement in this theoretical work is that not all spatial frequencies (ie sizes) are equally critical in visibility studies. In the special case of the Secchi disk it is shown that the modulation transfer approach and the radiative transfer approach converge. This is due to a rapid decrease in the MTF and a flattening in the high frequency region due to attenuation of transmitted contrast. The MTF theory applied in this work provides a different perspective from which a new understanding about underwater visibility issues can be achieved, and from which new applications of Secchi disk visibility to target identification and field instrumentation can be envisioned.

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